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DEVELOPMENT OF A MATHEMATICS MODULE ON INFINITE LIMITS OF ALGEBRAIC FUNCTIONS TO ENHANCE STUDENTS' CRITICAL THINKING SKILLS

Deny Hadi Siswanto[a*], Kintoko[b], Tarso[c], Rohmanur Mala[d], Rif'an Efendi[e]

[a] Teacher Professional Education, denyhadi.2024@student.uny.ac.id, Yogyakarta State University, Indonesia
[b] Mathematics Education, kintoko@upy.ac.id, PGRI Yogyakarta University, Indonesia
[c] Education Management, tarso.2024@student.uny.ac.id, Yogyakarta State University, Indonesia
[d] Education Management, rohmanurmala.2024@student.uny.ac.id, Yogyakarta State University, Indonesia
[e] Education Management, rifanefendi.2024@student.uny.ac.id, Yogyakarta State University, Indonesia
[*] Correspondence

ABSTRACT

This study aims to develop a mathematics module on the infinite limit material of algebraic functions to improve critical thinking skills of phase-F students at SMA Muhammadiyah 1 Sleman. The study used the ADDIE development model which includes five stages: Analysis, Design, Development, Implementation, and Evaluation. Data were collected through material and media expert validation questionnaires, student response questionnaires, and pretest and posttest tests. The validation results showed that the module was classified as very valid, with an average score of 71 from material experts and 103 from media experts. The student response questionnaire produced a practicality score of 72.6, indicating that the module was practical to use. The effectiveness test was carried out using the normality test and paired sample t-test using SPSS-25, which showed a significant increase (p <0.05) in student learning outcomes. The average pretest score of 58.22 increased to 84.27 in the posttest, with an n-Gain value of 0.64 which is classified as moderate. This module also improved scores on all indicators of students' critical thinking. Thus, the developed e-module is declared valid, practical, and effective in improving students' conceptual understanding and critical thinking skills.

***Keywords**: ADDIE, Module, Mathematics, Infinite Limit, Critical Thinking Skills*

Abstrak

Penelitian ini bertujuan untuk mengembangkan modul matematika pada materi limit tak hingga fungsi aljabar untuk serta untuk meningkatkan kemampuan berpikir kritis siswa fase-F di SMA Muhammadiyah 1 Sleman. Penelitian menggunakan model pengembangan ADDIE yang mencakup lima tahapan: Analisis, Desain, Pengembangan, Implementasi, dan Evaluasi. Data dikumpulkan melalui angket validasi ahli materi dan media, angket respons siswa, serta tes pretest dan posttest. Hasil validasi menunjukkan bahwa modul tergolong sangat valid, dengan rata-rata skor 71 dari ahli materi dan 103 dari ahli media. Angket respons siswa menghasilkan skor kepraktisan sebesar 72,6, menunjukkan modul praktis digunakan. Uji efektivitas dilakukan dengan uji normalitas dan paired sample t-test menggunakan SPSS-25, yang menunjukkan adanya peningkatan signifikan ($p < 0,05$) pada hasil belajar siswa. Nilai rata-rata pretest sebesar 58,22 meningkat menjadi 84,27 pada posttest, dengan nilai n-Gain sebesar 0,64 yang tergolong dalam kategori sedang. Modul ini juga meningkatkan skor pada seluruh indikator berpikir kritis siswa. Dengan demikian, e-modul yang dikembangkan dinyatakan valid, praktis, dan efektif dalam meningkatkan pemahaman konsep dan kemampuan berpikir kritis siswa.

Kata Kunci: ADDIE, Modul, Matematika, Limit Takhingga, Kemampuan Berpikir Kritis

1. INTRODUCTION

In the context of 21st-century education, critical thinking is recognized as one of the essential skills students must possess in order to compete and adapt to the increasingly complex global challenges [1]. This ability is

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crucial not only in academic settings but also in everyday life, where individuals are expected to analyze information, evaluate situations, and make sound decisions based on data and logic [2]. Within mathematics education, critical thinking plays a vital role, as students are not only required to perform mechanical calculations but also to comprehend concepts, construct logical arguments, and apply these concepts to real-world problems [3]. One particular topic in mathematics that demands high levels of critical thinking is the concept of infinite limits in algebraic functions. This material is often challenging for high school students due to its abstract nature, requiring deep conceptual understanding of limits and the skill to manipulate various algebraic forms in the context of unbounded behavior.

The topic of infinite limits in algebraic functions is fundamental in senior high school mathematics, serving as a foundation for learning calculus [4]. It not only lays the groundwork for understanding derivatives and integrals but also bridges earlier mathematical concepts with more advanced topics. However, classroom observations indicate that many students struggle to grasp this conceptually [5]. This difficulty is attributed not only to the inherent complexity of the topic but also to the lack of instructional materials that facilitate thorough understanding. Existing resources such as textbooks and conventional modules often emphasize formulas and procedural steps, with limited opportunities for students to explore the underlying meanings and develop reflective thinking [6]. Consequently, students tend to rely on rote memorization rather than conceptual understanding, making it difficult for them to solve problems that require reasoning and generalization.

To address these issues, numerous studies have shown that systematically and innovatively developed learning modules can serve as effective solutions [7]. Learning modules that adopt a cognitive-developmental approach can stimulate critical thinking by presenting structured content, varied practice problems, and thought-provoking questions [8]. A well-designed module does more than convey information it invites students to reflect on their thought processes, question the rationale behind concepts, and connect different mathematical ideas [9]. In the context of infinite limits, a module that integrates critical thinking strategies can help students build strong conceptual foundations, broaden their understanding of the applications of limits, and cultivate an analytical and reflective mindset in solving complex mathematical problems.

Critical thinking skills do not emerge instantly but must be nurtured and developed through appropriate and continuous learning experiences. This necessitates instructional approaches that emphasize idea exploration, conceptual reflection, and both independent and collaborative problem-solving [10]. Learning modules designed with such strategies offer meaningful and engaging learning experiences that encourage students to actively participate in the thinking process. These modules become more than just instructional aids they serve as tools for developing reasoning skills, testing hypotheses, and formulating justifiable logical arguments [11]. In learning infinite limits, modules that incorporate higher-order thinking tasks, case studies, and exploratory activities will help students see connections between concepts and their broader applications.

Moreover, the development of modules must consider students' diverse learning styles and varying levels of ability [12]. An effective module should accommodate these differences by providing varied activities, adaptive explanations, and learning contexts that relate to students' real-life experiences. The integration of scientific approaches and active learning strategies can enhance students' curiosity, strengthen understanding through hands-on experience, and facilitate the transfer of knowledge to new situations. Such modules also support teachers in designing more differentiated, student-centered instruction [13]. In the case of infinite limits, a contextualized approach embedded in the module can transform a difficult topic into one that is more accessible, engaging, and applicable.

Additionally, the use of modules as self-directed learning materials offers advantages in terms of flexibility and learning efficiency. Modules allow students to study independently outside class hours, revisit material they find difficult, and develop autonomous learning habits [14]. This is crucial for fostering independent learning, a hallmark of 21st-century education. Through modules, students learn not only to solve problems but also to understand the cognitive processes behind the solutions. Therefore, the development of a mathematics module on infinite limits that focuses on enhancing critical thinking is of vital importance [15]. Such a module can serve as a strategic tool to improve both the quality of mathematics instruction and students' overall academic achievement.

Based on the discussion above, it can be concluded that the development of a critical thinking-based mathematics module on the topic of infinite limits in algebraic functions is a pressing need in today's educational landscape [16]. A systematically, structurally, and contextually developed module is expected to address the challenges teachers and students face in learning mathematics. Beyond improving students' understanding of infinite limits, the module aims to foster and develop critical thinking as a core 21st-century competence [17]. Hence, this research focuses on the development of a mathematics module that not only facilitates the learning process but also serves as a strategic medium to cultivate students' critical thinking abilities, preparing them to face real-world challenges and contribute to future scientific advancement.

2. LITERATURE REVIEW

2.1 Learning Modules

Learning modules are a type of instructional material either in printed or digital format designed systematically and structurally to facilitate both guided and independent learning. These modules are developed with the intention of enabling students to learn progressively in accordance with predetermined learning objectives. According to Kevin et al. [18], a module is a self-contained learning package comprising a single unit of instruction, specifically designed to allow students to study independently and achieve specific outcomes. A typical learning module includes key components such as learning objectives, instructional content, learning activities, practice exercises, and assessments [19]. With this structure, students are provided the opportunity to construct their own understanding, deepen their knowledge incrementally, and cultivate autonomous learning skills.

The advantages of learning modules lie in their flexibility and their capacity to support differentiated instruction. Modules enable students with varying learning abilities to progress at their own pace while allowing teachers to offer more personalized guidance [20]. In the context of 21st-century education, which emphasizes active learning and higher-order thinking skills, modules developed using innovative approaches such as Problem-Based Learning (PBL), Project-Based Learning (PjBL), and scientific inquiry can have a positive impact on students' cognitive engagement. An effective learning module does not merely transmit information; it also encourages exploration, discussion, reflection, and the active construction of knowledge [21]. Thus, it is highly relevant to the development of critical thinking skills in mathematics instruction.

2.2 Infinite Limits

Infinite limits are a fundamental concept in calculus that relate to the behavior of a function as the input variable approaches infinity or when the limit itself tends toward infinity. This concept is foundational for understanding advanced topics such as derivatives, improper integrals, and function analysis. According to Siswanto and Kuswantara [22], infinite limits help students examine how functions grow or decrease without bound and are instrumental in understanding asymptotic behavior and function trends near the boundaries of a domain. However, for many high school students, the concept of infinite limits poses a significant challenge due to its abstract nature and the lack of real-world contextualization. As a result, students often resort to memorizing formulas and procedures without truly grasping the underlying meaning of the concept.

The difficulty in mastering infinite limits is further exacerbated by the limited use of visual approaches, real-life applications, and instructional materials that foster conceptual exploration. Therefore, the development of targeted learning modules focusing on infinite limits is necessary to bridge the gap in students' understanding. Modules that are systematically designed using problem-solving strategies, graphical illustrations, and reflective activities can support students in gradually building conceptual comprehension [23]. Consequently, such modules not only enhance students' mastery of the subject matter but also improve their critical thinking abilities particularly in analyzing function graphs, evaluating infinite growth patterns, and solving limit-related problems in both mathematical and real-world contexts.

2.3 Critical Thinking

Critical thinking refers to the ability to analyze, evaluate, and synthesize information in order to make logical and responsible decisions. According to Ennis [24], critical thinking encompasses skills such as interpretation, analysis, evaluation, inference, explanation, and self-regulation. In the context of education, critical thinking is essential as it enables students to engage deeply with content, assess arguments critically, and solve complex problems. Specifically in mathematics education, critical thinking empowers students not only to arrive at solutions but also to understand the processes involved, verify the correctness of their answers, and make connections between various mathematical concepts [25]. This skill becomes increasingly crucial in the 21st century, where individuals are expected to think logically and make decisions based on data and evidence.

To foster critical thinking, instructional design must encourage higher-order cognitive engagement. Learning modules serve as an effective medium for this purpose, especially when structured around approaches that promote exploration, real-world problem analysis, and student self-reflection [26], [27]. Modules that incorporate guiding questions, case studies, and investigative activities provide students with structured opportunities to practice critical thinking progressively. In mathematics instruction, such approaches are particularly effective for topics like infinite limits, where students are challenged to comprehend abstract concepts, evaluate multiple solution strategies, and justify their chosen methods with logical reasoning. In this way, critical thinking becomes not merely an instructional goal, but an integral component of the learning process itself.

3. METHOD

This study is a type of research and development (R&D) that employs the ADDIE instructional design model developed by Branch [28]. The model consists of five main phases: Analysis (identifying students' needs and characteristics), Design (creating the initial design of the product), Development (producing the instructional module), Implementation (field implementation of the product), and Evaluation (assessing the product's effectiveness and efficiency). Each stage is conducted systematically to ensure that the resulting product aligns with the learning objectives and the actual needs of students. The data collection techniques were tailored to the instruments employed in the study. To assess the validity of the developed module, validation questionnaires were administered to subject-matter experts and media experts using a 5-point Likert scale. In addition, the practicality of the module was measured through student response questionnaires focusing on ease of use and perceived usefulness, also utilizing the same scale.

In the data analysis process, a descriptive statistical approach was used to interpret the results of the questionnaires completed by experts and students. This provided quantitative insights into the feasibility and acceptability of the module. Furthermore, to examine the module's effectiveness in more detail, a normality test was first conducted to ensure that the data met distribution assumptions, followed by a paired sample t-test to determine whether there were statistically significant differences between the pretest and posttest scores. These statistical analyses were conducted using SPSS-25 to ensure accuracy and efficiency. Additionally, the normalized gain (n-Gain) score was calculated based on the formula from Hake [29] to determine the extent of improvement in students' learning outcomes and critical thinking skills based on the aspects outlined by Ennis [24] after using the module. This analysis provided valuable insights into the degree of influence or contribution the module had on significantly enhancing students' conceptual understanding during the learning process.

4. RESULTS AND DISCUSSION

4.1. Analysis

This study produced a module on the infinite limit material of algebraic functions that aims to improve critical thinking skills of phase F students. This module was developed using the ADDIE model which includes five stages, namely analysis, design, development, implementation, and evaluation. Based on the results of observations and interviews with phase-F mathematics teachers at SMA Muhammadiyah 1 Sleman, it is known that the learning process still relies on Student Worksheets which only contain a summary of the material and practice questions, so it is considered less interactive. The material discussed in the module includes infinite limits of algebraic functions, which overall consists of 44 lesson hours. However, the module developed is focused on the scope of only 8 lesson hours. The learning process is carried out twice a week with a duration of 2 x 45 minutes per meeting. The survey results also revealed common mistakes that students often make in understanding this material.

Table 1. Initial Survey Results

Relationship	Results				
Learning Process	Lecture	Proyek	Giving Questions	Culture Based	
	50%	31%	13%	6%	
Common Mistakes	Infinite limits of algebraic functions		Function derivative	Composition Function	
	60%		28%	12%	
Use of Teaching Materials	Module	Worksheets	Hand Out	Book	Comik
	68%	11%	9%	7%	5%

Based on the findings, the mathematics learning process is dominated by the lecture method at 50%, followed by project-based learning at 31%, questioning at 13%, and a culture-based approach at only 6%. In terms of common mistakes made by students, 60% of students made mistakes in the infinite limit material of algebraic functions, followed by function derivatives at 28%, and function composition at 12%. As for the use of teaching materials, most teachers use modules at 68%, then worksheets at 11%, handouts at 9%, books at 7%, and comics as teaching materials are only used at 5%. These findings indicate the dominance of traditional learning methods and the need for increased interactivity and improvement of teaching materials to overcome student difficulties, especially in the infinite limit material.

4.2. Design

The design of this research and development includes the design of the e-module and its research design. The module design process begins with the collection of infinite limit materials of algebraic functions. In addition, the researcher also compiled a questionnaire to measure validity and practicality, as well as test questions to assess student learning achievement. Furthermore, the module display is designed using the PowerPoint application, as shown in the following figure.

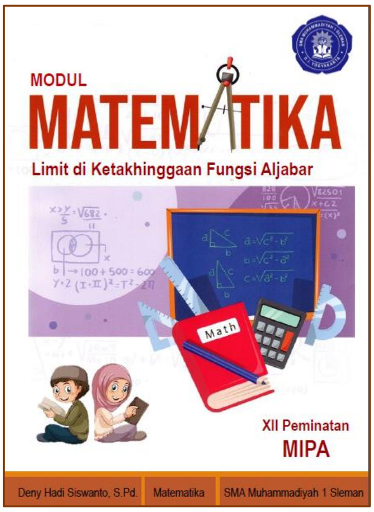

Figure 1. Cover

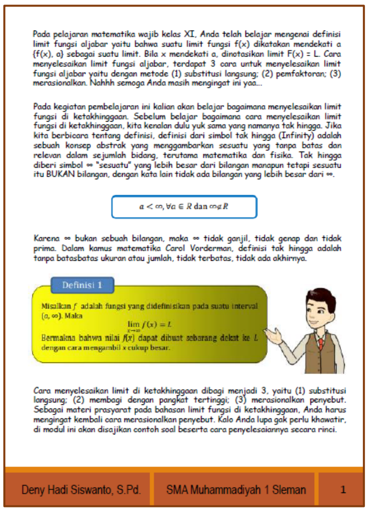

Figure 2. Material 1

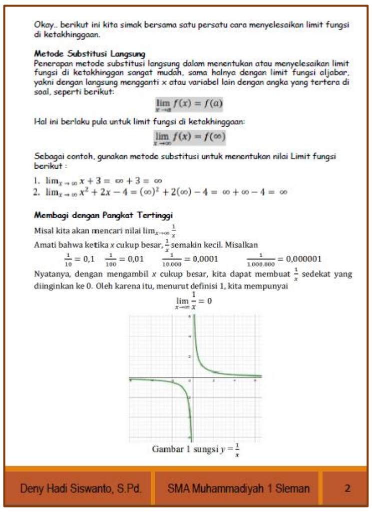

Figure 3. Material 2

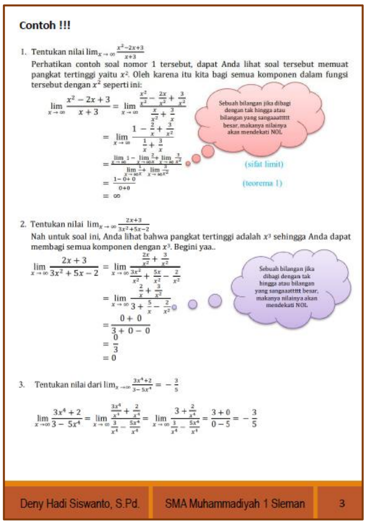

Figure 4. Example Question 1

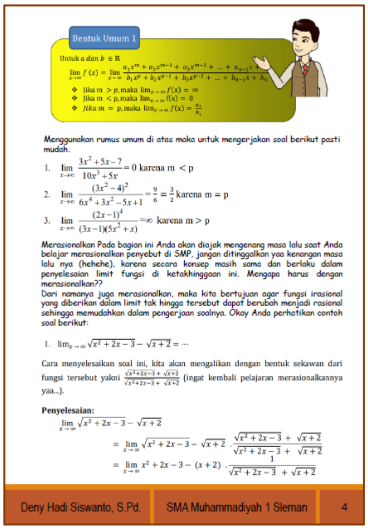

Figure 5. Material 3

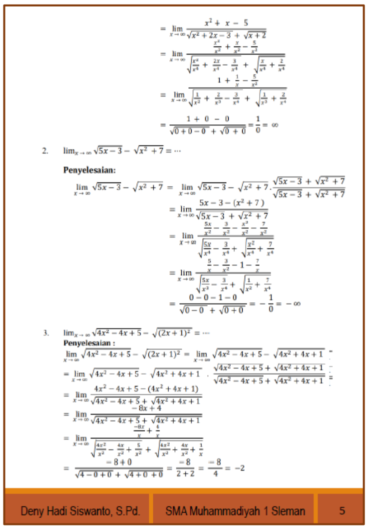

Figure 6. Example Question 2

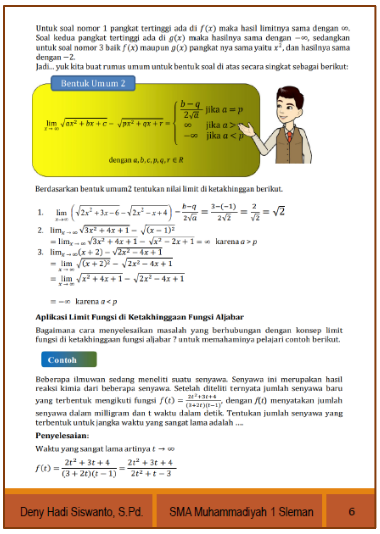

Figure 7. Material 4

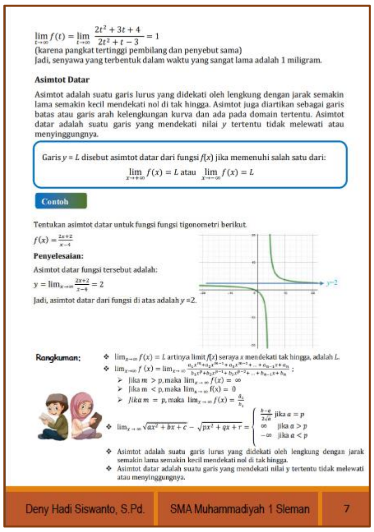

Figure 8. Example Question 3

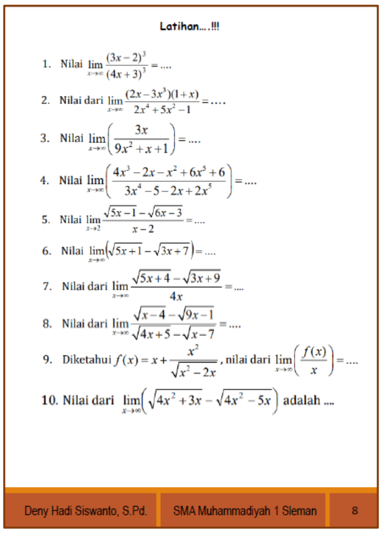

Figure 9. Practice Questions

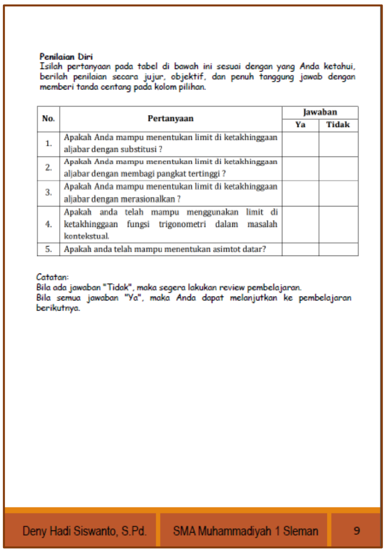

Figure 10. Self-Assessment

This study was designed to validate the process and technical aspects of implementing the module in the field. The study was conducted in class F phase of SMA Muhammadiyah 1 Sleman in two mathematics learning sessions. In the first meeting, an initial test was conducted to measure students' initial critical thinking skills before they used the module to study the material and work on practice questions. In the second meeting, students took the test and provided responses through a questionnaire response.

4.3. Development

The module that has been designed using PowerPoint is then converted into PDF format and printed for use. The module undergoes a validation process by material experts and media experts, with the validated aspects covering the material and media aspects of the module, aiming to assess the level of validity of each aspect based on the percentage and criteria that have been set in the research method. The validation sheet of the material expert is divided into three assessment aspects, namely the feasibility of content, language, and feasibility of presentation, where the aspect of content feasibility consists of 15 statements, the aspect of language consists of 5 statements, and the aspect of feasibility of presentation consists of 5 statements. Each statement has a maximum score of 5, so that the total maximum score of all statements is 75. The results of the assessment by the material expert are presented in Table 2 below.

Table 2. Material Expert Assessment

Aspect	Expert I	Expert II	Expert III
Content Suitability	23	24	22
Language	24	23	25
Presentation Suitability	23	25	23
Total Score	70	72	70
Average		71	

Based on the table above, the material in the e-module obtained an average score of 71 which according to the formula and criteria in the research method is included in the very valid category. Meanwhile, validation by media experts includes three aspects of assessment, namely content feasibility, presentation feasibility, and language feasibility. The aspect of content feasibility consists of 5 statements, presentation feasibility includes 5 statements, and graphic feasibility consists of 12 statements. Each statement has a maximum score of 5, so that the total maximum score is 110. The results of the assessment by media experts are presented in Table 3 below.

Table 3. Media Expert Assessment

Aspect	Expert I	Expert II	Expert III
Content Feasibility	23	24	23
Presentation Feasibility	22	23	24
Graphics Feasibility	57	58	55
Total Score	102	105	102
Average		103	

Based on the table above, the material in the module obtained an average score of 103 which, based on the formula and criteria in the research method, is categorized as very valid. After the assessment, the e-module was then improved according to the suggestions of material experts and media experts.

4.4. Implementation

After the module meets the validity standards based on evaluations from material experts and media experts, the e-module can be applied in the classroom learning process. Evaluation of the practicality and effectiveness of the e-module in the study can be done after this stage is passed. The practicality of the module is measured through observations of student responses obtained from filling out the questionnaire. The student response questionnaire was filled out by Phase-F students. This questionnaire consists of 17 statements covering four aspects of assessment, with a maximum score of 5 for each statement. The results of the student response questionnaire scores can be seen in Table 4 below.

Table 4. Student Response Questionnaire

Aspect	Total Average of each Aspect
Interest	17,3
Material	12,2
Language	17,1
Presentation	26
Total score	72,6

Based on the table above, the total student response score of 72.6 shows that the mathematics module on the infinite limit material of algebraic functions is categorized as practical based on the practicality test.

4.5. Evaluation

The evaluation stage is the final phase in research using the ADDIE model and aims to ensure that the developed module has good quality and is suitable for use in learning. The evaluation process begins with giving students a pretest to assess their critical thinking skills before taking part in learning using the module on the topic of calculating infinite limits of algebraic functions. After the pretest, students who were the subjects of the trial were given a posttest to determine the improvement in their learning outcomes after using the module. The effectiveness of the e-module was analyzed by comparing the results of the pretest and posttest using the paired sample t-test statistical test. Before testing was carried out, a normality test was first carried out with the help of the SPSS-25 program to ensure that the pretest and posttest data were normally distributed. The results of this analysis are the basis for determining the effectiveness of the developed module in improving student learning outcomes.

Table 5. Normality Test Results

Tests of Normality						
	Kolmogorov-Smirnov[a]			Shapiro-Wilk		
	Statistic	df	Sig.	Statistic	df	Sig.
Pretest	.145	29	.200*	.964	29	.230
Posttest	.185	29	.133	.952	29	.143

*. This is a lower bound of the true significance.
a. Lilliefors Significance Correction

Based on the table above, it can be seen that the Shapiro-Wilk significance value for the pretest data is 0.230 > 0.05 and for the posttest data is 0.143 > 0.05. Thus, it can be concluded that both data are normally distributed. After that, the researcher conducted a paired sample t-test to compare the test results with the help of SPSS-25 for Windows software. The results of the paired sample t-test are presented in Table 6 below.

Table 6. Paired Sample T-Test Results

Paired Samples Test									
		Paired Differences							
		Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference Lower	Upper	t	df	Sig. (2-tailed)
Pair 1	Pretest-Posttest	2681.86	1353.31	251.30	3169.63	2167.08	10.63	28	.000

Based on the table above, it can be seen that the significance value (Sig. 2-tailed) of 0.000 is smaller than the significance limit of 0.05. This shows that there is a significant difference between the pretest and posttest scores. In other words, there was an increase in the score after the treatment was given. This increase indicates that the use of the infinite limit module in algebraic functions has proven effective in improving learning outcomes. To find out how much the students' thinking skills have increased in more detail, calculations using the n-Gain formula are used, with the results presented as follows.

$$\langle g \rangle = \frac{S_{post} - S_{pre}}{SMI - S_{pre}} = \frac{85,00 - 58,15}{100 - 58,15} = \frac{26,85}{41,85} = 0,64$$

Based on the calculation results above, it can be seen that the average pretest and posttest values in the N-Gain calculation are 0.64 in the moderate category. The increase can be seen in the following figure.

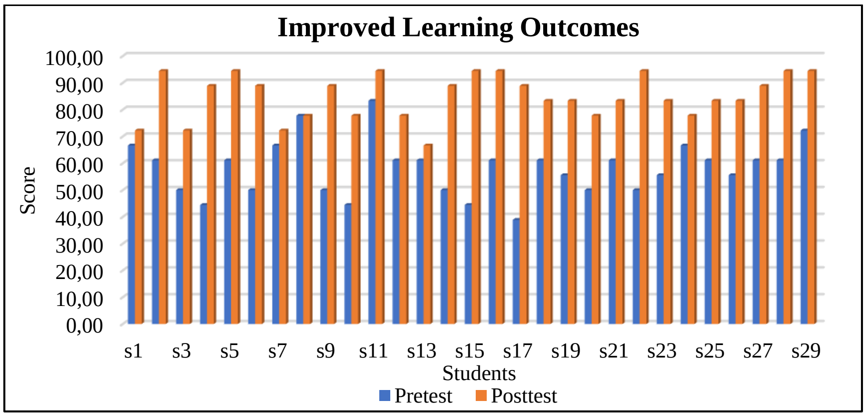

Figure 11. Improved Learning Outcomes

Based on the pretest and posttest score data from 29 participants, the maximum pretest score was 83.33 and the minimum score was 38.89, while the maximum posttest score reached 94.44 and the minimum score was 66.67. The average pretest score was around 58.22, while the average posttest score increased to 84.27. The highest increase between the pretest and posttest scores occurred in the 2nd participant (s2) and several other participants who achieved an increase of up to 33.33 points, indicating a significant increase in learning outcomes after using the module. Meanwhile, the increase in critical thinking skills can be seen in the following figure.

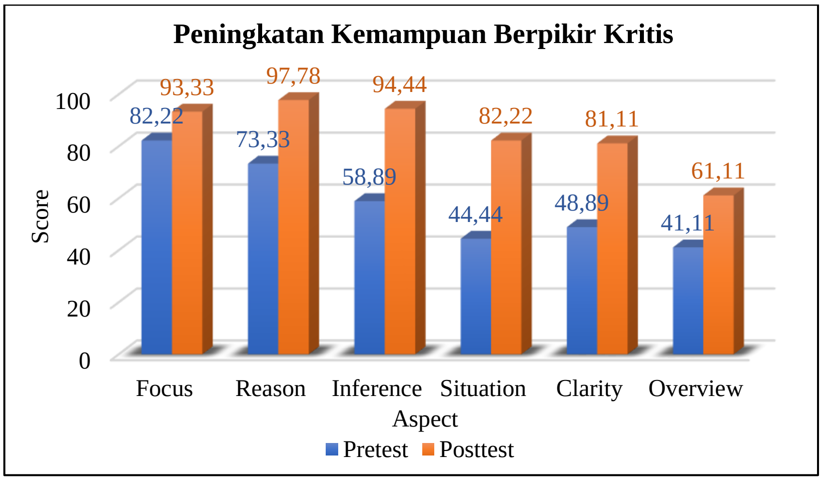

Figure 12. Improving Critical Thinking Skills

Based on the data in the table, there was an increase in the score of each indicator from pretest to posttest. The Focus indicator increased from 82.22 to 93.33, while Reason experienced a significant increase from 73.33 to 97.78. The Inference indicator also showed a spike from 58.89 to 94.44. Furthermore, the Situation indicator increased from 44.44 to 82.22. The Clarity indicator increased from 48.89 to 81.11, and the Overview indicator increased from 41.11 to 61.11. Overall, all indicators showed an increase in value which reflects positive developments after the implementation of the posttest.

4.6. Discussion

This study produced a PowerPoint-based instructional module in PDF format, focusing on the topic of infinite limits of algebraic functions. The module is intended to enhance the critical thinking skills of Phase F students at SMA Muhammadiyah 1 Sleman. The development of the module followed the systematic ADDIE model, which includes the stages of Analysis, Design, Development, Implementation, and Evaluation. Initial survey results indicated that the learning process was still predominantly lecture-based (50%), while interactive approaches such as questioning strategies and culturally responsive teaching were minimally applied. These findings underscore the urgency of developing more interactive instructional materials that can actively engage students especially for topics such as infinite limits of algebraic functions, which are generally perceived as difficult.

Based on validation results by subject matter experts and media experts, the developed module was found to have a very high level of validity, with average scores of 71 for the content aspect and 103 for the media aspect. The validation process assessed several critical components, including content appropriateness, language use, presentation, and graphic elements that support readability and visual clarity. These findings are consistent with research by Wahyuni et al. [30] and Siswanto [31], which emphasized that the validity of a module is a key indicator for ensuring that the material aligns with learning objectives and the intended basic competencies. Moreover, a module that is valid in both substance and presentation tends to be more effective in delivering concepts clearly and systematically.

During the implementation stage, students responded positively to the module, with an average practicality score of 72.6, which falls into the practical category. This supports the findings of Tarso et al [32] and Suryatama et al. [33], who argued that practical instructional materials enable students to learn independently and more effectively by minimizing confusion in understanding the concepts. Students' high responses in the areas of presentation and language indicate that the module's visual design and narration greatly supported their comprehension. This demonstrates that the use of visual media such as PowerPoint whether in print or digital modules can significantly enrich the learning experience [34], [35].

Furthermore, the pretest and posttest results showed a significant increase in student scores, with the average pretest score at 58.22 and the posttest score at 84.27. A paired sample t-test revealed a significance value of

0.000 ($p < 0.05$), indicating a statistically significant difference between the two scores. The n-Gain score was calculated at 0.64, which falls within the medium category. These results align with the findings of Hake [29], who noted that medium to high n-Gain scores indicate that a learning intervention has a positive impact on students' conceptual understanding. In other words, this module successfully enhanced students' learning outcomes in a significant way.

Upon further analysis, improvements in students' critical thinking skills were evident across all indicators. The most substantial gains were observed in the Reasoning and Inference indicators, suggesting that the module was effective not only in strengthening content mastery but also in developing students' analytical and logical abilities. According to Facione [36], critical thinking involves reasoning, inference, and clarification all of which were reflected in the indicators assessed in this study. Thus, the improvements in these aspects confirm that the instructional approach used in the module aligns well with the principles of critical thinking development.

In conclusion, this study makes a valuable contribution to the development of effective and practical instructional materials. The infinite limit module for algebraic functions developed in this research has been proven to be valid, practical, and effective in enhancing the critical thinking skills of Phase F students. These findings reinforce previous research by Astiwi et al. [37] and Syah et al. [38], which stated that the integration of innovative learning media in the form of modules can improve learning outcomes and higher-order thinking skills. Therefore, the use of such modules is highly recommended for broader application in mathematics education, particularly for topics that are traditionally considered difficult by students.

5. CONCLUSION

This study successfully developed an infinite limit module of algebraic functions with the main objective of improving critical thinking skills of phase-F students. The module was developed using the ADDIE model which includes five systematic stages: analysis, design, development, implementation, and evaluation. Validation by material and media experts showed very valid results, with average scores of 71 and 103 respectively. In its implementation, the e-module showed high practicality (average student response 72.6) and significant effectiveness in improving learning outcomes, as evidenced by a paired sample t-test with a significance of 0.000 and an increase in n-Gain of 0.64 (moderate category). This module has also been shown to be able to develop indicators of critical thinking skills, especially in the aspects of reasoning and inference. The implications of this study are the need for a shift in learning strategies from lectures to participatory approaches based on innovative media such as e-modules and the importance of strengthening assessments of students' critical thinking skills explicitly, with evaluations that not only measure the final results but also the thinking process during learning.

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