



DIFFERENTIATED ASSESSMENT AND DIGITAL VISUALIZATION FOR ENHANCING STUDENTS' REASONING IN CONIC SECTIONS LEARNING

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ABSTRACT

This descriptive qualitative study, supplemented by quantitative data, evaluated advanced mathematics instruction and assessment on conic sections for Grade XII (Phase F) students. The participants included one teacher and 12 students. Data were collected through semi-structured interviews, direct observations, and document analysis. Findings revealed uneven conceptual mastery (mean: 74.17), with hyperbolas identified as the most challenging topic. Students struggled to transform algebraic equations into graphical models, where only 50.00% accurately identified essential characteristic points. Furthermore, performance on contextual problem-solving (mean: 65.83) was significantly lower than on procedural tasks (mean: 80.00). The integration of dynamic digital visualization enhanced visual comprehension, achieving a higher mean score (82.08) compared to written assessments (73.75). Continuous and differentiated assessment demonstrated a positive learning progression, increasing mean scores from diagnostic (64.58) to final tests (79.17) with 83.33% mastery. Consequently, evaluating conic sections requires continuous, differentiated approaches combining technology-assisted visualization and contextual tasks to foster higher-order mathematical reasoning.

Keywords: Conic Sections, Learning Assessment, Digital Visualization, Differentiated Assessment

1. INTRODUCTION

The implementation of the Kurikulum Merdeka at the senior high school level introduces Advanced Mathematics in Phase F, designed to deepen students' mathematical understanding through higher-order reasoning. Mathematics learning at this level no longer focuses solely on rote memorization and procedural symbolic manipulation; rather, it demands the formation of robust conceptual understanding, spatial reasoning, and inter-domain mathematical connections [1]. Mastering abstract concepts serves as a foundational pillar for Grade XII students preparing for higher education and applications in science and technology disciplines [2]. However, achieving these curricular goals faces complex operational challenges, particularly concerning students' readiness to simultaneously integrate various abstract mathematical representations [3]. Consequently, evaluating the instructional processes and assessment systems implemented by teachers in the classroom becomes crucial to ensure that the targeted Phase F Learning Outcomes (Capaian Pembelajaran) are realized effectively.

One of the most challenging domains within Phase F Advanced Mathematics is analytic geometry, specifically the topic of conic sections encompassing circles, ellipses, parabolas, and hyperbolas. Conic sections require students to precisely connect algebraic representations in general and standard forms with two-dimensional geometric graphical representations [4]. This interconnectedness necessitates a firm grasp of essential parameters such as the centre, vertex, focus, directrix, and asymptotes that define the unique characteristics of each curve [5]. Learning conic sections is often perceived as difficult because students must master dual-translation capabilities namely, transforming non-standard algebraic equations into standard forms through completing the square while simultaneously interpreting the geometric implications of each algebraic parameter [6]. This complex conceptual structure makes the conic sections topic a key indicator for

measuring the extent to which Grade XII students' visual-spatial and algebraic reasoning skills have harmoniously developed.

Empirical evidence from the field reveals significant disparities in students' mastery of conic sections. Diagnostic studies and instructional evaluations demonstrate that the majority of students remain trapped in rote procedural problem-solving and experience a drastic decline in performance when confronted with contextual problem-solving tasks or graphical translations [7]. Specifically, hyperbolas and ellipses frequently present the highest degree of difficulty due to the complex determination of foci and asymptotes from general equations [8]. Common fundamental errors include students' inability to identify key coordinates, procedural mistakes in completing the square, and failure to identify relevant conic models within real-world problem contexts [9]. The gap between procedural execution and contextual reasoning indicates that traditional learning evaluations focused exclusively on final written test scores fail to comprehensively map students' learning barriers.

In addressing the visual-spatial complexity of conic sections, the integration of digital technology such as interactive flat panels and dynamic geometry software plays a strategic role in the learning and assessment process. Dynamic visualization enables students to interactively manipulate algebraic equation parameters and directly observe resulting graphical transformations on screen, thereby bridging abstract algebraic concepts to concrete visual forms [10]. The presence of digital tools has been proven to enhance student engagement and assist them in verifying algebraic calculations through graphical proof independently [11]. Nevertheless, the utilization of digital technology does not automatically guarantee deep conceptual understanding unless accompanied by analytical and reflective assessment designs [12]. Digital tools in evaluation are frequently restricted to visual verification without adequately exploring students' ability to explain the underlying mathematical meaning behind graphical changes.

In addition to technological integration, the effectiveness of advanced mathematics instruction relies heavily on the presence of continuous and differentiated assessment systems. Diversity in initial algebraic proficiency and learning paces among Grade XII students dictates that teachers should refrain from relying on a single end-of-unit assessment (single-shot assessment) that ignores individual developmental dynamics [13]. Diagnostic assessments at the beginning of instruction are necessary to map prerequisite algebraic knowledge, followed by periodic formative assessments to detect misconceptions early on [14]. Implementing differentiated assessments integrated with remedial and enrichment activities provides struggling students the opportunity to catch up, while simultaneously facilitating high-achieving students to explore higher-order thinking tasks. Therefore, a responsive assessment framework adapted to individual learning styles and levels of conceptual mastery represents an urgent necessity in classroom instruction.

Based on these dynamics and issues, this study aims to comprehensively evaluate the instruction and assessment of Phase F Grade XII Advanced Mathematics on the topic of conic sections. The evaluation focuses on four main pillars: conceptual mastery of basic algebra-geometry, algebraic-to-graphical transformation skills, contextual problem-solving, and the effective integration of digital technology alongside differentiated assessment. The novelty of this research lies in its holistic evaluation approach based on data triangulation synthesizing teacher perceptions, classroom interaction dynamics, and assessment documentation to generate a complete picture of students' learning barriers and achievements in conic sections. The evaluation findings are expected to offer theoretical and practical contributions for mathematics educators in designing instructional strategies, selecting appropriate visualization media, and constructing continuous assessment instruments oriented toward enhancing students' mathematical reasoning in the Kurikulum Merdeka era.

2. LITERATURE REVIEW

2.1. Conic Sections in Advanced Mathematics Education

The study of conic sections comprising circles, ellipses, parabolas, and hyperbolas occupies a central position in upper secondary mathematics, particularly within advanced analytic geometry frameworks such as Phase F of the Kurikulum Merdeka. Mastering conic sections requires students to construct dual-directional connections between symbolic algebraic expressions and two-dimensional geometric properties [15]. Rather than relying solely on computational algorithms, understanding conics demands spatial reasoning to locate essential structural features, including centers, vertices, foci, directrices, and asymptotes [16]. This integration of spatial-visual thinking with algebraic fluency serves as a critical bridge for high school students preparing for STEM-oriented tertiary education, where advanced calculus and vector geometry prominently

feature these curves [17]. Consequently, the topic acts as a robust indicator of a student's overall mathematical maturity and ability to navigate multi-representational tasks.

Despite its pedagogical significance, empirical literature consistently highlights conic sections as one of the most challenging domains in secondary mathematics due to its high cognitive load and structural abstraction. Students frequently encounter severe hurdles when attempting to convert general quadratic equations into standard forms through algebraic procedures such as completing the square [18]. While foundational curves like circles are relatively accessible, more complex conics specifically hyperbolas and non-standard rotated parabolas frequently trigger persistent misconceptions regarding orientation, focal distance, and asymptotic behaviors [19]. These difficulties stem from an overreliance on rote formula memorization rather than an intuitive understanding of the locus-based definitions of curves, resulting in a sharp decline in student performance when visual supports are absent or when equations are presented in non-routine configurations.

2.2. Learning Assessment in Secondary Mathematics

Educational assessment in upper secondary mathematics has progressively shifted from measuring basic procedural recall toward evaluating higher-order thinking skills (HOTS), conceptual flexibility, and real-world problem-solving capacities. Contemporary curricular reform frameworks, including Indonesia's Kurikulum Merdeka, emphasize that learning assessment must capture students' cognitive processes rather than merely evaluating final numerical answers [20]. In the context of advanced geometry and algebra, effective evaluation designs must probe whether learners can formulate mathematical models, select appropriate strategies, and interpret mathematical results within contextual environments [21]. Authentic assessment tasks such as performance-based modeling, open-ended problem solving, and reflective explanations provide deeper insights into how students structure their mathematical reasoning when confronted with complex scenarios [22].

However, traditional classroom evaluation practices frequently suffer from an alignment gap, relying predominantly on routine, single-shot written tests administered at the conclusion of an instructional unit [23]. Research indicates that while students may score satisfactorily on procedural items by applying memorized algorithms, their performance deteriorates significantly when items demand contextual interpretation or non-routine problem-solving [24]. This discrepancy reveals that conventional summatively-heavy testing often obscures underlying conceptual flaws, such as an inability to translate word problems into geometric parameters. To overcome these limitations, scholars advocate for comprehensive evaluation systems that synthesize diagnostic, formative, and summative mechanisms, thereby offering a continuous and ecologically valid record of student progress throughout the learning trajectory.

2.3. Digital Visualization in Mathematics Instruction

The incorporation of digital visualization tools including dynamic geometry environments (e.g., GeoGebra) and Interactive Flat Panels (IFP) has transformed the teaching and learning of abstract mathematical constructs. Dynamic visualization allows students to manipulate algebraic parameters interactively via sliders or touch interfaces and instantly observe the corresponding transformations on geometric graphs [25]. By making abstract mathematical relationships visible and manipulable, digital tools alleviate cognitive load and bridging between symbolic, graphical, and geometric representations [26]. Meta-analytic investigations demonstrate that technology-assisted instruction generally yields positive medium-to-large effect sizes on student achievement, particularly in topics that heavily rely on spatial reasoning and dynamic curve transformations [27].

Nevertheless, modern research emphasizes that digital technology acts as a cognitive instrument whose efficacy depends strictly on how it is integrated into pedagogical and assessment designs. While dynamic visual tools significantly increase student engagement and provide immediate graphical verification, operating software independently does not automatically yield deep conceptual understanding [28]. Without guided analytical prompts, reflective questioning, and structured tasks, students may passively manipulate software tools without internalizing the underlying mathematical principles governing the visual changes [29]. Therefore, literature suggests that digital visualization should be embedded directly into assessment practices, requiring learners to explain, justify, and connect visual evidence back to formal algebraic logic rather than using digital media solely as a graphical shortcut.

2.4. Differentiated Assessment in the Inclusive Classroom

Differentiated assessment is rooted in the recognition that students enter the classroom with diverse prior knowledge, cognitive paces, learning styles, and readiness levels. Influenced by differentiated instruction

frameworks, differentiated assessment rejects a "one-size-fits-all" approach, advocating instead for flexible evaluation strategies tailored to accommodate individual learning trajectories [30]. In advanced secondary mathematics, where variances in prerequisite algebraic proficiency can severely impact a student's ability to grasp new abstract geometric topics, differentiated assessment provides an essential mechanism for maintaining equity and instructional responsiveness [31]. By systematically gathering diagnostic and formative data, teachers can identify specific learning gaps and adapt subsequent tasks to match varying degrees of student readiness.

In practical application, differentiated assessment operates as an ongoing, iterative cycle involving baseline diagnostic mapping, targeted formative feedback, and adaptive post-assessment interventions. Struggling learners are provided with targeted scaffolding, foundational remediation, and scaffolded representational tasks, whereas advanced learners are offered complex enrichment tasks and open-ended contextual investigations [32]. Empirical evidence demonstrates that implementing continuous differentiated assessment significantly improves aggregate mastery rates over time, reducing the performance gap between low-performing and high-performing students [33]. In complex domains such as conic sections, this approach ensures that assessment functions not as a passive filtering mechanism, but as an active, supportive engine that drives conceptual growth and mathematical confidence across all learner profiles.

3. METHOD

This study employed a descriptive qualitative approach supplemented by supporting quantitative data to evaluate advanced mathematics instruction on conic sections in Grade XII (Phase F). The research subjects comprised one mathematics teacher and 12 Grade XII students currently studying conic sections, including circles, ellipses, parabolas, and hyperbolas. Data collection was carried out through three main techniques: semi-structured interviews with the teacher to explore evaluation strategies and constraints, direct classroom observations of student learning dynamics, and document analysis. The collected documents included diagnostic assessment sheets, formative and summative test results, and assessment summaries derived from the use of interactive digital medi. Qualitative data analysis applied an interactive model comprising data reduction, data display in the form of narrative descriptions and comparative tables, and conclusion drawing [34]. Quantitative data from student test scores were processed using descriptive statistics to calculate mean scores, mastery percentages, and the proportion of student competencies per indicator [35]. Data trustworthiness was maintained through method and source triangulation by cross-referencing teacher interview results, classroom observation field notes, and student assessment documentation. This approach ensured that evaluations of conceptual understanding, algebraic-to-graphical transformation skills, contextual problem-solving, and digital technology integration were comprehensively analyzed and validated.

4. RESULTS AND DISCUSSION

4.1. Uneven Mastery of Fundamental Concepts of Conic Sections

The interview findings indicated that students' mastery of the fundamental concepts of conic sections was relatively uneven. The mathematics teacher explained that some Grade XII students were able to identify the characteristics of circles, ellipses, parabolas, and hyperbolas, whereas other students still experienced difficulties distinguishing the standard forms of their equations and relating algebraic expressions to the corresponding geometric characteristics. These difficulties became more apparent when students were required to determine the type of conic section from an equation without visual assistance. According to the teacher, students generally demonstrated better performance when learning materials were accompanied by graphical representations, concrete examples, and step-by-step explanations. However, when assessment items were presented primarily in symbolic or algebraic forms, several students had difficulty determining the centre, vertex, focus, axis, and other essential characteristics of a given conic section. The teacher therefore identified conceptual understanding as an important area requiring attention, particularly the ability to connect algebraic representations with geometric properties.

Table 1. Summary of Students' Mastery in Fundamental Concepts of Conic Sections

Indicator	Supporting Data
Students achieving mastery	8 students (66.67%)
Students not achieving mastery	4 students (33.33%)
Mean score	74.17
Highest score	92
Lowest score	58
Relatively easier topic	Circle

Relatively more difficult topic	Hyperbola
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The classroom observations supported the interview findings. Students appeared to follow worked examples relatively well, particularly when the teacher explicitly demonstrated how to identify the characteristics of each type of conic section. Nevertheless, several students became less confident when required to independently classify conic sections and explain the mathematical reasoning underlying their answers. Documentation of the formative assessment further confirmed this condition. Of the 12 students participating in the learning activities, 8 students (66.67%) achieved the predetermined mastery criterion for the conceptual identification indicator, while 4 students (33.33%) had not yet achieved the criterion. The mean score was 74.17, with the highest score of 92 and the lowest score of 58. Students demonstrated relatively stronger performance in identifying circles, whereas hyperbolas remained comparatively difficult. These findings indicate that the evaluation of conic-section learning should assess not only students' ability to obtain correct answers but also their conceptual understanding and their ability to establish relationships among equations, graphical representations, and geometric properties.

4.2. Difficulties in Transforming Equations into Graphical Representations

The interview findings revealed that students experienced difficulties in transforming algebraic equations into graphical representations of conic sections. The teacher reported that students were generally able to perform routine algebraic procedures when equations were already presented in standard form. However, their performance declined when they were required to transform non-standard equations into standard forms before constructing the corresponding graphs. This process required students to integrate several mathematical skills, including algebraic manipulation, completing the square, identifying coordinates, determining geometric parameters, and interpreting the resulting equation. The teacher further explained that common errors included mistakes in completing the square, incorrectly determining the centre of a circle or ellipse, misidentifying the vertex and opening direction of a parabola, and confusing the transverse and conjugate axes of a hyperbola. During the learning process, the teacher therefore frequently used guiding questions to help students identify relevant information contained in equations and connect algebraic procedures with geometric interpretations.

Table 2. Student Performance in Transforming Conic Section Equations into Graphical Representations

Assessment Aspect	Number Correct	Percentage
Determining the standard form	9	75.00%
Determining the centre/ vertex	8	66.67%
Determining the direction of the curve	7	58.33%
Determining important points	6	50.00%
Constructing an accurate graph	7	58.33%

The classroom observations were consistent with these interview findings. Students were more engaged when graphical representations were introduced, but several students still required teacher guidance to translate an algebraic expression into an accurate graphical model. Documentation of students' assessment sheets showed that only 7 of the 12 students (58.33%) achieved a score of at least 75 on the indicator concerning the transformation of equations into graphical representations. The detailed results demonstrated that students performed relatively well in determining standard forms, but their performance decreased when they were required to identify important points and construct accurate graphs. In particular, only 6 students (50.00%) were able to determine the relevant characteristic points correctly, while 7 students (58.33%) were able to produce an appropriate graphical representation. These findings demonstrate that graphical representation should be incorporated as an essential component of assessment rather than treated merely as an additional activity after algebraic manipulation. Evaluation should explicitly examine whether students can move fluently between symbolic, graphical, and geometric representations.

4.3. Lower Performance in Contextual Problem-Solving Compared with Procedural Tasks

The interview findings indicated that students performed better on procedural questions than on contextual and complex problem-solving tasks. The teacher explained that students tended to obtain satisfactory results when assessment questions followed patterns similar to those previously demonstrated during classroom instruction. In such situations, students could identify the appropriate formula and apply a familiar sequence of mathematical procedures. However, their performance decreased when they encountered contextual problems requiring them to independently determine which concept of conic sections was relevant to a particular situation. Such tasks required students not only to perform calculations but also to interpret information, identify mathematical relationships, construct an appropriate mathematical model, select a

strategy, and interpret the resulting solution. The teacher observed that several students immediately attempted algebraic calculations without first identifying the known information, unknown quantities, and mathematical relationships represented in the problem. Consequently, errors frequently occurred at the stage of problem interpretation rather than solely during mathematical computation.

Table 3. Student Achievement Metrics Across Varying Item Complexity Levels

Type of Assessment Item	Mean Score	Students Achieving Mastery
Concept identification	78.33	9 (75.00%)
Procedural problems	80.00	10 (83.33%)
Semi-contextual problems	71.67	8 (66.67%)
Contextual problems	65.83	6 (50.00%)
Complex problem-solving	62.50	5 (41.67%)

The classroom observations further demonstrated that students required more time and guidance when working on contextual and complex problems. Several students asked clarification questions before determining the appropriate conic-section model, while others attempted to use previously memorised procedures even when those procedures were not directly applicable to the given situation. Documentation of the assessment results demonstrated a difference between procedural and contextual performance. The mean score for procedural questions reached 80.00, with 10 students (83.33%) achieving mastery, whereas the mean score for contextual problems was 65.83, with only 6 students (50.00%) achieving mastery. The lowest performance was found in complex problem-solving tasks, for which the mean score was 62.50 and only 5 students (41.67%) achieved mastery. These findings suggest that assessment of conic-section learning should provide sufficient opportunities for students to demonstrate higher-order mathematical processes, including problem interpretation, mathematical modelling, strategic reasoning, justification, and interpretation of solutions, rather than relying predominantly on routine procedural exercises.

4.4. Use of Digital Technology Supports Evaluation but Remains Suboptimal

The interview findings showed that the mathematics teacher had incorporated digital technology into the learning and assessment of conic sections, particularly to support the visualisation of mathematical graphs. According to the teacher, digital graphical tools enabled students to observe more directly the relationship between changes in the parameters of an equation and changes in the corresponding graphical representation. This was considered particularly useful for conic sections because several characteristics, such as the centre, vertex, focus, axis, direction, and shape of a curve, can be difficult for students to visualise through symbolic representations alone. The teacher also observed that digital visualisation encouraged students to compare different equations and examine how modifications to mathematical parameters affected the resulting curves. Nevertheless, the interview indicated that the use of technology did not automatically guarantee conceptual understanding. Some students were able to manipulate the digital tool effectively but were still unable to explain the mathematical meaning of the graphical changes they observed. Therefore, technological use needed to be accompanied by analytical questions and reflective assessment tasks.

Table 4. Student Achievement and Independence in Digital Visualization-Assisted Assessment

Evaluation Data	Result
Students independently using digital media	10 students (83.33%)
Students requiring assistance	2 students (16.67%)
Mean score of visualisation-based assessment	82.08
Mean score of written assessment	73.75
Difference in mean scores	8.33 points

The classroom observations confirmed that digital visualisation generated greater student engagement during the exploration of conic sections. Students were able to examine several graphs, compare their characteristics, and verify their algebraic solutions through graphical representations. However, two students still required direct teacher assistance to operate the digital medium and interpret the resulting graphs. Documentation of the learning activities indicated that 10 of the 12 students (83.33%) were able to use the digital visualisation tool independently, while 2 students (16.67%) required additional assistance. The mean score obtained from visualisation-based assessment was 82.08, compared with 73.75 for written assessment, resulting in an 8.33-point difference. Although these data do not by themselves establish a causal relationship between technology use and achievement, the triangulated findings suggest that digital visualisation provided a useful learning

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environment for connecting algebraic and graphical representations. Consequently, evaluation should integrate technology-supported tasks while ensuring that students are required to explain, interpret, and justify the mathematical meaning of visual representations rather than merely manipulate digital graphs.

4.5. Need for More Continuous and Differentiated Assessment

The interview findings indicated that the existing assessment practices had not fully accommodated differences in students' prior knowledge and learning needs. The teacher explained that classroom evaluation involved a combination of exercises, assignments, quizzes, and written tests. However, the assessment results demonstrated substantial variation in students' mastery of conic-section concepts. Some students required reinforcement of fundamental concepts before they could successfully engage with more complex problems, whereas other students were already capable of solving higher-level tasks and therefore required enrichment activities. The teacher further indicated that differences in learning pace became particularly visible when students studied parabolas and hyperbolas, which required students to integrate algebraic transformations with geometric interpretation. These findings suggest that a single assessment administered at the end of a learning sequence may not provide sufficient information for identifying students' specific learning difficulties. A more systematic assessment process involving diagnostic, formative, remedial, and enrichment activities would provide more detailed information regarding individual learning progress.

Table 5. Progression of Student Achievement and Mastery Rates Across Continuous Assessment Stages

Evaluation Stage	Mean Score	Students Achieving Mastery	Students Not Achieving Mastery
Diagnostic assessment	64.58	5 (41.67%)	7 (58.33%)
Circle–Ellipse assessment	72.50	8 (66.67%)	4 (33.33%)
Parabola assessment	76.67	9 (75.00%)	3 (25.00%)
Hyperbola assessment	70.83	8 (66.67%)	4 (33.33%)
Final assessment	79.17	10 (83.33%)	2 (16.67%)

The classroom observations supported the need for continuous and differentiated assessment. Students with weaker initial conceptual understanding required more time and teacher scaffolding, while students who had already mastered the basic concepts were able to complete more complex tasks independently. Documentation of the assessment results demonstrated a positive progression from the diagnostic assessment to the final assessment. The mean score increased from 64.58 in the diagnostic assessment to 79.17 in the final assessment, while the proportion of students achieving mastery increased from 41.67% to 83.33%. Although this development indicates improved learning outcomes, two students remained below the mastery criterion in the final assessment. The findings therefore emphasise the importance of using assessment as an ongoing process for identifying learning needs, providing timely feedback, implementing remedial instruction, and designing enrichment activities. Such an approach can make the evaluation of Phase F advanced mathematics more responsive to differences in students' conceptual understanding, algebraic proficiency, graphical reasoning, and problem-solving abilities.

4.6. Discussion

The first finding indicates that students' mastery of fundamental conic-section concepts was uneven, with 8 of 12 students (66.67%) achieving mastery and a mean score of 74.17, while hyperbola was identified as the relatively more difficult topic. The interview and observation data suggest that the main difficulty was not merely computational, but concerned students' ability to connect algebraic equations with geometric characteristics such as centre, vertex, focus, and axis. This finding is consistent with the broader literature on mathematical visualisation, which emphasises that visual representations can help students understand abstract mathematical concepts and establish connections between different mathematical ideas [36], [37], [38]. The present finding therefore strengthens the argument that assessment of conic sections should move beyond the measurement of procedural correctness and explicitly examine conceptual connections among symbolic, graphical, and geometric representations. In this context, the lower performance on hyperbola-related concepts can be interpreted as an indication that students require more opportunities to construct and interpret representations rather than simply memorise the characteristics of each conic section. This interpretation is also compatible with research showing that GeoGebra-assisted mathematics learning can improve conceptual mathematical abilities, although the effectiveness depends on how the technology is incorporated into instruction [39], [40], [41]. The current study consequently extends previous findings by showing, through interview, observation, and assessment documentation, how conceptual difficulties appear specifically during the evaluation of Grade XII Phase F conic-section learning.

The second finding concerns students' difficulty in transforming equations into accurate graphical representations. The assessment documentation showed that although 75.00% of students could determine the standard form, only 50.00% could correctly determine important points, and 58.33% could construct an accurate graph. These results indicate a progressive decrease in performance as the task moved from relatively routine algebraic manipulation toward integrated symbolic and graphical reasoning. Such a pattern is important because conic sections inherently require students to coordinate algebraic and geometric representations. Previous research has demonstrated that external visualisations have a positive overall effect on mathematics learning, with a meta-analysis of 41 studies involving 10,562 students reporting a medium overall effect of visualisation interventions [42], [43]. More specifically, Zhang et al. reported a positive medium-to-large effect of dynamic GeoGebra visualisation on mathematics achievement, based on 19 effect sizes from 14 studies involving 1,334 participants [44], [45]. The present findings are therefore consistent with previous research in suggesting that visualisation can support mathematical learning, but they also highlight an important assessment issue: providing a graph is not sufficient unless students are required to construct, interpret, and explain the graph from an algebraic representation. Thus, the difficulty observed in the present study suggests that assessment should deliberately include bidirectional translation between equations and graphs rather than assessing these representations separately.

The third finding demonstrates a substantial difference between students' performance on procedural and contextual problem-solving tasks. Students achieved a mean score of 80.00 on procedural problems, whereas their mean score decreased to 65.83 on contextual problems and 62.50 on complex problem-solving tasks. This pattern suggests that students were relatively more successful when the required procedure was familiar but experienced greater difficulty when they had to interpret a situation, determine the appropriate conic-section model, select a strategy, and justify the solution. The finding is important because mathematical competence involves more than executing known procedures; students must also coordinate conceptual knowledge, representation, reasoning, and problem-solving strategies. Recent research on digital mathematics learning similarly indicates that dynamic visualisation can support students' awareness during task interpretation, strategic exploration, monitoring, and evaluation, particularly when technology is used as a cognitive instrument rather than merely as a graphing device [46], [47], [48]. The present results consequently suggest that the evaluation of conic sections should contain a greater variety of problem structures, including contextual and non-routine problems that require students to explain why a particular conic model is appropriate. This finding complements earlier research rather than simply reproducing it, because the present study identifies the discrepancy between procedural mastery and contextual problem-solving within a specific Grade XII Phase F classroom and confirms the pattern through classroom observations and assessment documentation.

The fourth finding concerns the use of digital technology to support the evaluation of conic sections. The study found that 10 of 12 students (83.33%) could use digital media independently, while the mean score on visualisation-based assessment was 82.08, compared with 73.75 on written assessment. Although these figures should not be interpreted as evidence of a causal effect, they indicate that digital visualisation created conditions in which students could more readily explore and compare graphical representations. This finding is strongly supported by previous meta-analytic evidence. Juandi et al. (2021), synthesising 36 effect sizes from 29 studies involving 2,111 students, reported a positive overall effect of GeoGebra on students' mathematical abilities, with an estimated Hedges' g of 0.96. Similarly, Yildiz & Arpacı [49] found a positive medium-to-large effect of GeoGebra as a dynamic visualisation tool, although the magnitude of the effect varied according to factors such as topic, treatment duration, and sample size. A recent systematic review further indicates that GeoGebra can support metacognitive processes through dynamic visualisation, real-time feedback, monitoring, and cross-representational checking, while also noting that evaluation is sometimes restricted to visual verification [50]. Therefore, the present finding reinforces the importance of combining digital visualisation with explanatory and reflective assessment so that students do not merely manipulate graphs but understand the mathematical relationships represented by them.

The fifth finding shows the importance of continuous and differentiated assessment in Phase F conic-section learning. The mean score increased from 64.58 in the diagnostic assessment to 79.17 in the final assessment, while the proportion of students achieving mastery increased from 41.67% to 83.33%. Nevertheless, two students remained below the mastery criterion at the end of the learning sequence, indicating that improvements in aggregate achievement do not necessarily mean that all students have reached comparable levels of conceptual and procedural understanding. The interview and observation findings further showed that students differed in the amount of scaffolding and time they required, particularly in topics involving

algebraic transformations and geometric interpretation. This supports an assessment approach in which diagnostic assessment is used to identify initial difficulties, formative assessment is used to monitor learning progress, and subsequent remedial or enrichment activities are adjusted according to individual needs. The finding also complements the literature on technology-supported visualisation, which suggests that the effectiveness of learning interventions can vary according to learner and instructional conditions rather than operating uniformly across contexts [41], [51]. Thus, the contribution of the present study lies not only in documenting an improvement in assessment results but also in demonstrating, through triangulated qualitative evidence, why differentiated and continuous assessment is necessary for conic-section learning: students enter the learning process with different levels of conceptual understanding and require different forms of feedback, scaffolding, remediation, and enrichment.

5. CONCLUSION

The study concludes that the evaluation of Grade XII Phase F Advanced Mathematics on conic sections revealed uneven conceptual mastery, difficulties in connecting algebraic equations with graphical representations, lower achievement in contextual and complex problem-solving, and differences in students' learning needs. Digital visualisation showed potential to support students' understanding, while the improvement from the diagnostic assessment to the final assessment indicated positive learning progress. Therefore, teachers are recommended to implement continuous and differentiated assessment through diagnostic, formative, remedial, and enrichment activities, complemented by digital visualisation and contextual problem-solving tasks that require students to explain, represent, reason, and justify their mathematical solutions.

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